TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 3 2019

Calculator Free

Discrete Random Variables, Binomial Distribution

STUDENT'S NAME SOLUTIONS

DATE: Thursday 16th May **TIME:** 25 minutes **MARKS:** 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser and formula sheet

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1. (5 marks)

The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{7}, \quad x = 1, 2, 3, 4, 5, 6, 7$$

(a) Determine the value of $E(X)$ [1]

$= \frac{n+1}{2}$ $E(X) = 4$

$= \frac{7+1}{2}$

The standard deviation of X is 2.

(b) Determine:

(i) $E(3X - 1)$ [2]

$= 3(4) - 1$

$= 11$

(ii) $Var(3 - 2X)$ [2]

$= (-2)^2(2)^2$

$= 16$

2. (7 marks)

The probability function $f(x)$ of a random variable X is defined as:

$$f(x) = \begin{cases} \frac{x}{6} & x = 0, 1 \\ \frac{1}{x} & x = 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Explain why x is not a Bernoulli random variable [1]

Does not represent two outcomes of an experiment - success/failure

(b) Give three reasons to prove that $f(x)$ defines a probability function of a discrete random variable. [3]

- x vaules are discrete
- sum of probabilities = 1
- All probabilities are positive

(c) Determine:

(i) $P(X = 2 \text{ and } X = 1)$ [2]

$= \frac{1}{2} + \frac{1}{6}$

$= \frac{2}{3}$

(ii) $P(X < 2)$ [1]

$= \frac{1}{6}$

3. (8 marks)

The probability distribution function of X is tabulated below.

x	1	2	3	4	5
$P(X=x)$	$k-0.2$	0.1	k	$k-0.3$	$k-0.2$

(a) Determine the value of k. [3]

$k-0.2+0.1+k+k-0.3+k-0.2=1$

$4k-0.6=1$

$4k=1.6$

$k=0.4$

(b) Determine:

(i) $P(X>1)$ [1]

$=1-0.2$

$=0.4$

(ii) $P(X\le 4\mid X>1)$ [2]

$=\frac{0.1+0.4+0.1}{0.8}$

$=\frac{0.6}{0.8}$ $=\frac{3}{4}$

(c) Calculate $E(X)$, the expected value of X. [2]

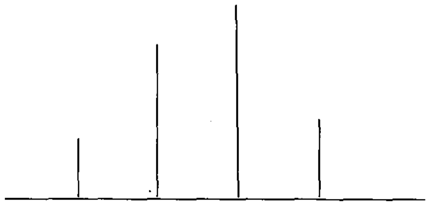
$E(X)=(1\times 0.2)+(2\times 0.1)+(3\times 0.4)+(4\times 0.1)+(5\times 0.2)$

$=3$

4. (4 marks)

This graph represents a binomial probability distribution.

The height of the first column is 0.027.



(a) State the value of n [1]

$n = 3$

(b) Calculate the probability of success for the binomial distribution. [3]

$P(X = 0) = {}^3C_0 \, p^0 (1-p)^{3-0}$

$0.027 = 1 \times 1 \times (1-p)^3$

$\sqrt[3]{0.027} = 1 - p$

$0.3 = 1 - p$

$p = 0.7$

5. (3 marks)

A particular binomial distribution consists of n trials and the probability of a successful outcomes on each trial is p. If the experiment has an expected value of 36 and a standard deviation of 3, determine the values of n and p.

$E(X) = np = 36$

$sd = \sqrt{np(1-p)} = 3$

$np(1-p) = 9$

$36(1-p) = 9$

$p = \frac{3}{4}$

$n = 48$

TRINITY COLLEGE

Year 12 Methods Units 3,4
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Calculator Allowed
Discrete Random Variables, Binomial Distribution

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6. (3 marks)

The following table shows a discrete probability distribution. If the expected value of X is 2, determine the values of a and b.

x	0	1	2	3	4
$P(X = x)$	a	$a + b$	0.2	0.3	0.1

$$2a + b + 0.6 = 1 \quad (1)$$

$$a + b + 0.4 + 0.9 + 0.4 = 2 \quad (2)$$

SOLVE ON CAS : $a = 0.1$

$b = 0.2$

7. (7 marks)

James is a soccer player and the probability that he scores a goal when he takes a shot is 0.3. In a soccer game he takes five shots on goal.

(a) Calculate the following probabilities.

(i) He scores a goal in his first and last shot but missed the goal with the other attempts. [1]

$0.3 \times 0.7^3 \times 0.3 = 0.0309$

(ii) He scores one goal. [2]

$X \sim B(5, 0.3)$

$P(X = 1) = 0.3602$

(iii) He scores at least one goal. [2]

$P(X \geq 1) = P(1 \leq X \leq 5)$

$= 0.8319$

(b) In another game, how many shots on goal does James expect to have to make so that he has at least a 90% chance of scoring at least two goals. [2]

binomialCDf $(2, x, x, 0.3)$

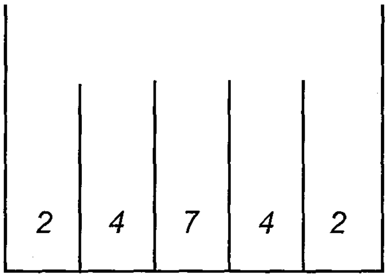
$n = 11 \quad = 0.8870$

$n = 12 \quad = 0.9150$

$\therefore \quad n = 12$

8. (9 marks)

In a game, a player rolls two balls (a Red and a Blue) down an inclined plane so that each ball finally settles in one of five slots and scores the number of points allotted to that slot as shown in the diagram below:



It is possible for both balls to settle in one slot and it may be assumed that each slot is equally likely to accept either ball. The player's score is the sum of the points scored by each ball.

(a) If the discrete random variable X is the score obtained by the two balls, complete the following probability distribution table for X. [4]

x	4	6	8	9	11	14
$P(X=x)$	0.16	0.32	0.16	0.16	0.16	0.04

	2	4	7	4	2
2	4	6	9	6	4
4	6	8	11	8	6
7	9	11	14	11	9
4	6	8	11	8	6
2	4	6	9	6	4

A player pays 10 cents for each game and receives back a number of cents equal to their score.

(b) If the discrete random variable Y is the number of cents won per game, complete the following probability distribution table for Y. [2]

y	–6	–4	–2	–1	1	4
$P(Y=y)$	0.16	0.32	0.16	0.16	0.16	0.04

(c) Calculate the players expected gain or loss per 50 games. [3]

$E(Y) = -2.4$ c / game

$= -120$ c / 50 games

$\therefore$ expected to lose \$1.20

9. (9 marks)

Eddie has decided that every time he does a multiple choice test, he will complete the test by selecting answers completely at random, for every question in the test.

(a) Eddie does a multiple choice test with 10 questions, each of which has 4 possible answers.

(i) If C represents the number of correct answers Eddie gets in this test, state the appropriate probability distribution for C. [1]

$C \sim B(10, 0.25)$

(ii) What is the probability that Eddie will score at least 30% on this test? [2]

$P(X \geq 3) = P(3 \leq x \leq 10)$

$= 0.4744$

(b) Eddie completes 20 such multiple choice tests over the course of a year.

(i) If T represents the number of tests in which Eddie gets at least 3 answers correct, state the probability distribution of T. [1]

$T \sim B(20, 0.4744)$

(ii) What is the probability that Eddie scores at least 30% on at least 40% of the multiple choice tests he completes in a year? [2]

40% of 20 = 8

$P(T \geq 8) = P(8 \leq T \leq 20)$

$= 0.8128$

(c) In each of his 6 years of secondary school, Eddie does 20 of these multiple choice tests.

(i) If Y represents the number of years in which Eddie scores at least 30% on at least 40% of the multiple choice tests he completes in that year, state the probability distribution of Y. [1]

$Y \sim B(6, 0.8128)$

(ii) What is the probability that Eddie scores at least 30% on at least 40% of the tests he completes in at least 50% of his years of secondary school? [2]

50% of 6 = 3

$P(Y \geq 3) = P(3 \leq Y \leq 6)$

$= 0.9867$